MATH 504 HOMEWORK 6

Due Monday, October 29.

A poset $\mathbb{P}$ is *separative* if whenever $p \not\leq q$, there is $r \leq p$ such that $r \perp q$.

Problem 1. *Let M be a model of ZFC and $\mathbb{P} \in M$ be a separative poset. Let $p, q \in \mathbb{P}$ and let $\dot{G}$ be the canonical $\mathbb{P}$-name for the generic filter. (I.e. $\dot{G} = \{\langle \check{r}, r\rangle \mid r \in \mathbb{P}\}$.) Show that $p \leq q$ iff $p \Vdash q \in \dot{G}$. Which direction uses separativity?*

Problem 2. *Let $\mathbb{P}, \mathbb{Q}$ be two posets and $\pi : \mathbb{P} \to \mathbb{Q}$ be a map such that:*

1. *for $p, q \in \mathbb{P}$, if $p \leq q \to \pi(p) \leq \pi(q)$,*
2. *for $p \in \mathbb{P}$ and $r \in \mathbb{Q}$, if $r \leq \pi(p)$, then there is some $q \leq p$ such that $\pi(q) \leq r$*

Show that if G is $\mathbb{P}$-generic over a model M, then the upward closure of π"G i.e. $H := \{r \in \mathbb{Q} \mid \exists p \in G(\pi(p) \leq r)\}$ is a $\mathbb{Q}$-generic filter over M. Then show that $M[H] \subset M[G]$.

Such a π as in the above problem is called a *projection*.

Problem 3. *For a cardinal κ, let $Add(\omega, \kappa)$ be the poset of all partial functions $f : \kappa \times \omega \to \{0, 1\}$ such that $\mathrm{dom}(f)$ is finite, ordered by reverse inclusion. Show that there is a projection from $Add(\omega, \kappa)$ to $Add(\omega, 1)$.*

Problem 4.
1. *Show that $Add(\omega, 1)$ is separative.*
2. *Show that $Add(\omega, \kappa)$ is separative.*

Problem 5. *Let M be a model of ZFC and let G be $Add(\omega, \kappa)$-generic over M. In $M[G]$, define $f^* = \bigcup_{f \in G} f$. Show that f^* is a function with domain $\kappa \times \omega$; i.e. $f^* : \kappa \times \omega \to \{0, 1\}$ is a total function.*